MODULE - 3

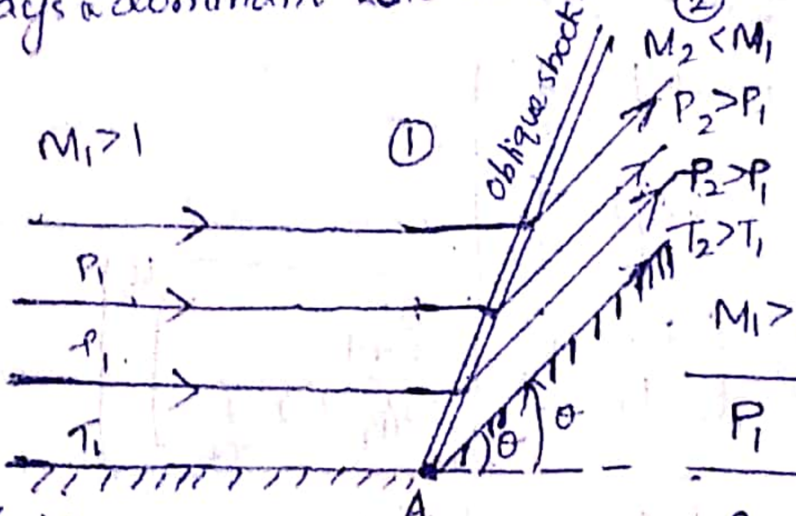
In case of supersonic flow, a compression front will be formed across which there will be abrupt changes in fluid properties. Normal shock wave is produced with 90° to the surface.

Normal shock wave is a oblique shock with inclined angle $\pi/2$.

A shock wave which is making an oblique angle to the surface is called oblique shock wave.

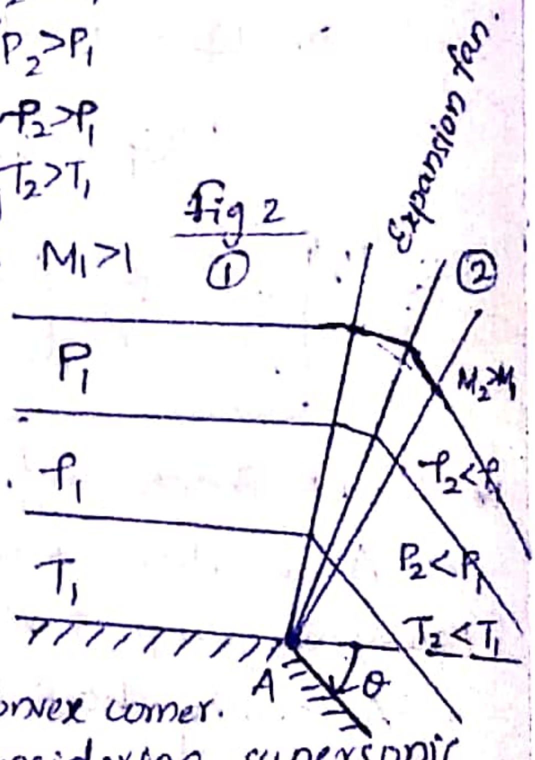
the oblique shock occurs when a supersonic flow turned into itself. When it is turned away from itse. If is called expansion fan. Oblique shock and expansion fan plays a dominant role in flowfields involving supersonic flow.

Fig 1:



(a) Concave corner

Fig 2



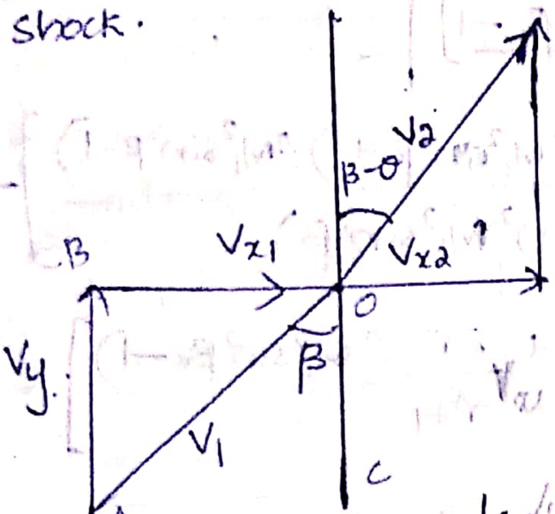
(b) Convex corner.

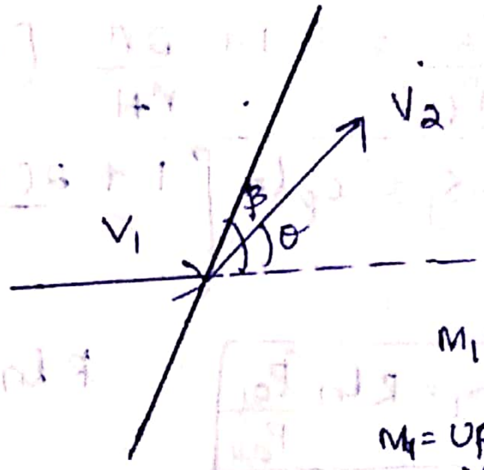
In the figure 1, we are considering supersonic flow over a wall with a corner A. Wall is turned θ° upward. Here a concave corner will be formed. All the flow should be tangent to the wall. After crossing the wave it is deflected uniformly upward making an angle θ with the horizontal. Flow turned to itself produces oblique shock. After crossing the Oblique shock Mach number gets reduced but the rest of the properties (T, P, ρ) increases.

In the figure a, wall is deflected downwards at an angle θ. Flow should be tangent to the wall. So it will be deflected downward. A convex corner is formed. The flow is moving away from supersonic flow. Therefore expansion fan is formed. Here M ↑ses and rest of the properties (P, T, ρ) ↓ses. (V↑ses)

* <u>Oblique Shock Relations:</u>

We are considering a normal shock & superimposing this normal shock with a uniform velocity V_y. V_y is ∥el to the normal shock in the flow field of normal shock. Now we obtain a oblique shock.





$M_1 = \frac{V_1}{a_1}$

M_1 = Upstream Machnumber

where, β = Shock angle / wave angle

$V_{x1} > V_{x2}$, so the inclination of the flow to the shock; ahead of the shock & after the shock are different.

The flow turns towards the shock. The angle θ by which the flow turns towards the shock is called the flow deflection angle and is +ve.

$$V_1 = \sqrt{V_{x1}^2 + V_y^2}$$

$$\tan\beta = \frac{V_{x1}}{V_y}$$

From Δle (where 'V_1' is the resultant velocity upstream of the shock)

$$\beta = \tan^{-1}\left(\frac{V_{x1}}{V_y}\right)$$ (where 'β' is the inclined angle to the shock).

The rotation of the flow field by an angle 'β' results in V_1 in horizontal direction. The shock inclined at angle 'β' to the incoming supersonic flow is called the <u>Oblique shock</u>.

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* Mach number normal to the shock is M_{n1} (M_{n1} is a component of M_1) $\therefore M_{n1} = M_1 \sin\beta$ & $M_{n2} = M_2 \sin(\beta-\theta)$.

Replace M_1 by $M_1 \sin\beta$ in all normal shock relations:

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2\sin^2\beta}{(\gamma-1)M_1^2\sin^2\beta+2}} \longrightarrow (A).$$

$$\boxed{\frac{T_2}{T_1} = 1 + \frac{2(\gamma-1)(\gamma M_1^2\sin^2\beta+1)(M_1^2\sin^2\beta-1)}{(\gamma+1)^2 M_1^2\sin^2\beta}} = \frac{h_2}{h_1} = \frac{a_2^2}{a_1^2}$$

$$\boxed{\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left[M_1^2\sin^2\beta-1\right]}$$

$$S_2-S_1 = C_p \ln\left[1 + \frac{2(\gamma-1)(\gamma M_1^2\sin^2\beta+1)(M_1^2\sin^2\beta-1)}{(\gamma+1)^2 M_1^2\sin^2\beta}\right] -$$

$$\boxed{S_2-S_1 = R\ln\frac{P_{01}}{P_{02}}} \qquad R\ln\left[1 + \frac{2\gamma}{\gamma+1}(M_1^2\sin^2\beta-1)\right].$$

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2}M_1^2\sin^2\beta}{\gamma M_1^2\sin^2\beta - \left(\frac{\gamma-1}{2}\right)}$$

M_{n2} is the normal mach no. behind the shock wave.

$$M_{n2} = M_2 \sin(\beta-\theta)$$

$$\therefore \boxed{M_2 = \frac{M_{n2}}{\sin(\beta-\theta)}}$$

where $$\boxed{M_n^2 = \frac{M_1^2\sin^2\beta + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2\sin^2\beta - 1}} \longrightarrow \text{from normal shock relations.}$$

- $M_1 \sin\beta$ should be supersonic.
  $M_1 \sin\beta \geq 1$.

  $\therefore \beta = 90°$.

- So the maximum value of $\beta = \pi/2$
- The possible value of wave angle.

$$\boxed{\sin^{-1}\left(\frac{1}{M_1}\right) \leq \beta \leq \pi/2}$$

Consider the first figure.

$$\boxed{\tan\beta = \frac{V_{x1}}{V_y}} \rightarrow ①$$

$$\boxed{\tan(\beta-\theta) = \frac{V_{x2}}{V_y}} \rightarrow ②$$

Divide eqn ② by eqn ①:

$$\boxed{\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{V_{x2}}{V_{x1}}} \rightarrow ③$$

From Continuity Eqn:

$$\rho_1 V_{x1} = \rho_2 V_{x2}$$

$$\therefore \boxed{\frac{V_{x2}}{V_{x1}} = \frac{\rho_1}{\rho_2}} \rightarrow ④$$

Sub eqn ④ in eqn ③

$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{\rho_1}{\rho_2}$$

$$\boxed{\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{(\gamma-1)M_1^2\sin^2\beta + 2}{(\gamma+1)M_1^2\sin^2\beta}}$$ (Sub Eqn Ⓐ in ④)

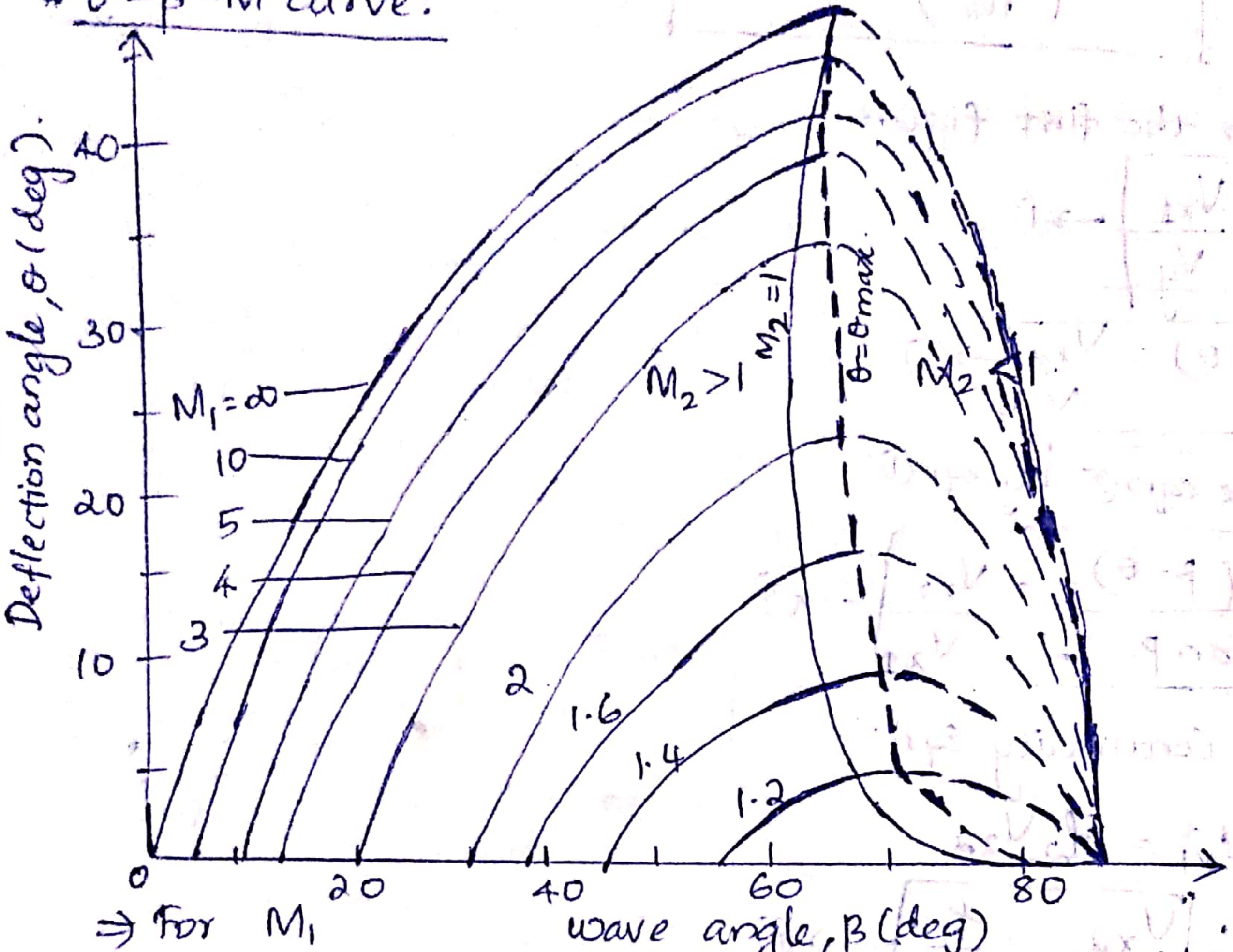
This is the implicit relation b/w θ & β for a given M_1. We can convert this to the explicit relationship.

Explicit relationship provides elaborated results.

$$\boxed{\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}\right]}$$

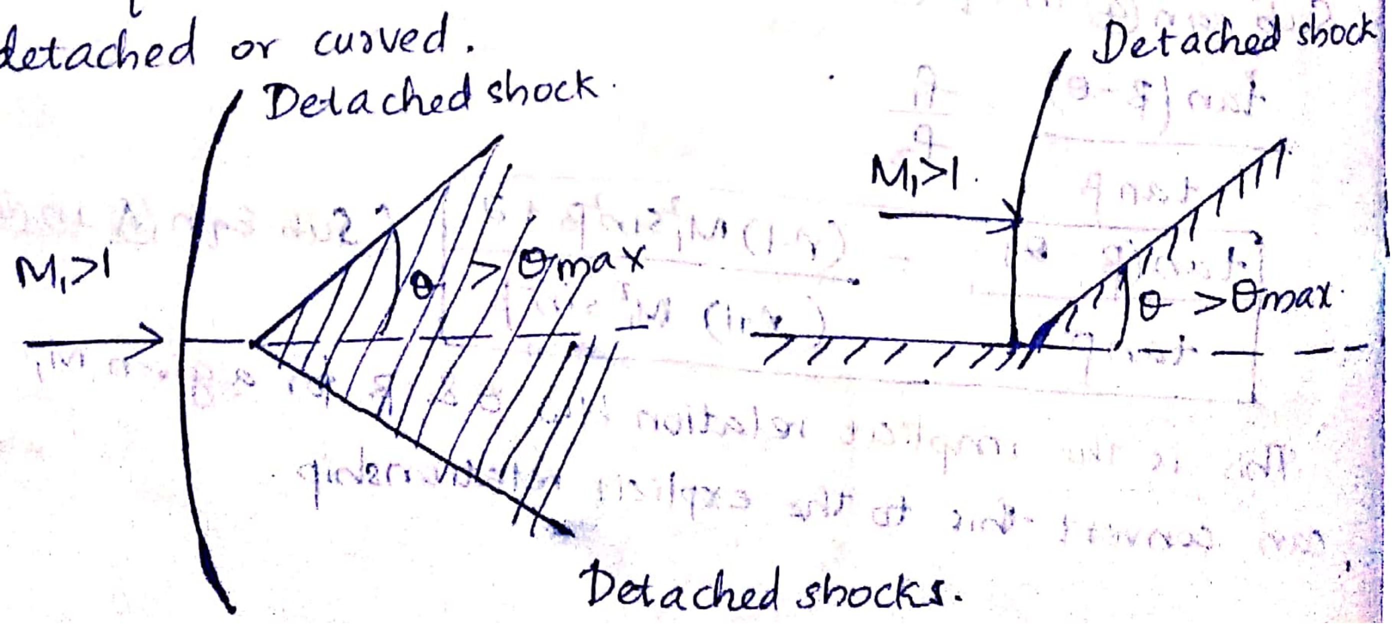
This is called $\theta-\beta-M$ relation.

* $\theta-\beta-M$ curve:



⇒ For M_1

(I) When $\theta > \theta_{max}$ then no solution is possible for straight oblique shock wave. In that particular case curve is detached or curved.



Detached shocks.

(II) when $\theta < \theta_{max}$, 2 possible solutions are there. i.e., it will produce 2 different wave angles (β).

If β value is larger, then that particular value is known as strong shock solution & if it is smaller then it is weak shock solution.

In the case of strong shock soln, the flow behind the shock will be subsonic & in case of weak shock solution, the flow behind the shock will be supersonic

(III) when $\theta = 0$

then $\beta = \frac{\pi}{2}$ (from eqn of $\tan\theta$).

Then the shock will be normal shock otherwise we can tell that shock disappeared.

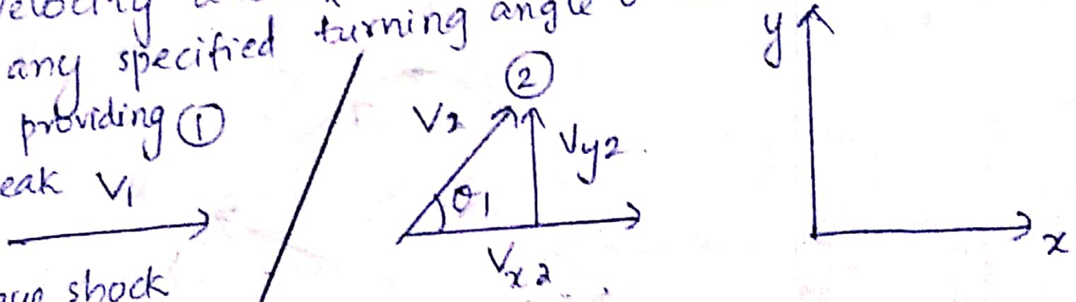
* Shock Polar:

→ The graphical representation of oblique shock properties.

→ The x-y plane is physical plane.

→ Let V_{x1}, V_{y1}, V_{x2} & V_{y2} be the x and y components of flow velocity ahead of and behind the shock.

→ for any specified turning angle 'θ' there are 2 possible shock angles & providing strong & weak solutions.

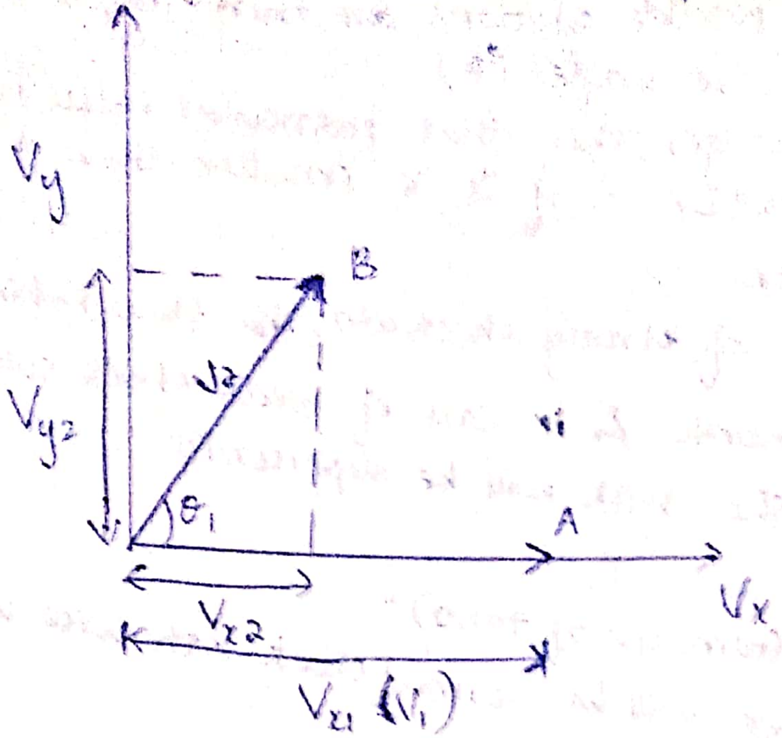


→ An oblique shock with upstream velocity V_1 in the xy-cartesian coordinate system is shown in fig. It has the velocity components V_x & V_y in the downstream field.

→ Oblique shock.

→ Let us represent the Oblique shock field in a plane with V_x & V_y as the axes. This plane is called the hodograph plane.

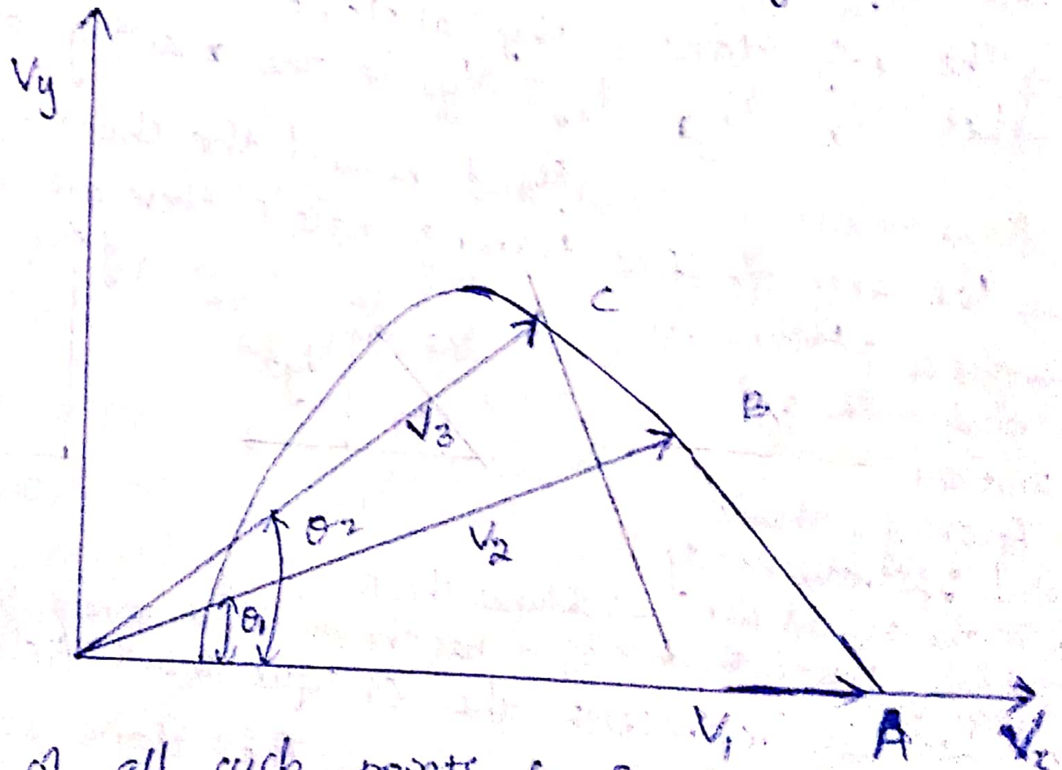
Graph:



→ Point 'A' represents region 1 in xy plane and B represents the region 2 ie, A – flow field ahead of the shock

B – flow field behind the shock

→ When θ_1 ↑ses shock becomes more stronger; V ↓ses



→ The locus of all such points for θ values from zero to θ_{max} representing all the possible velocities behind the shock. It is called the shock polar.

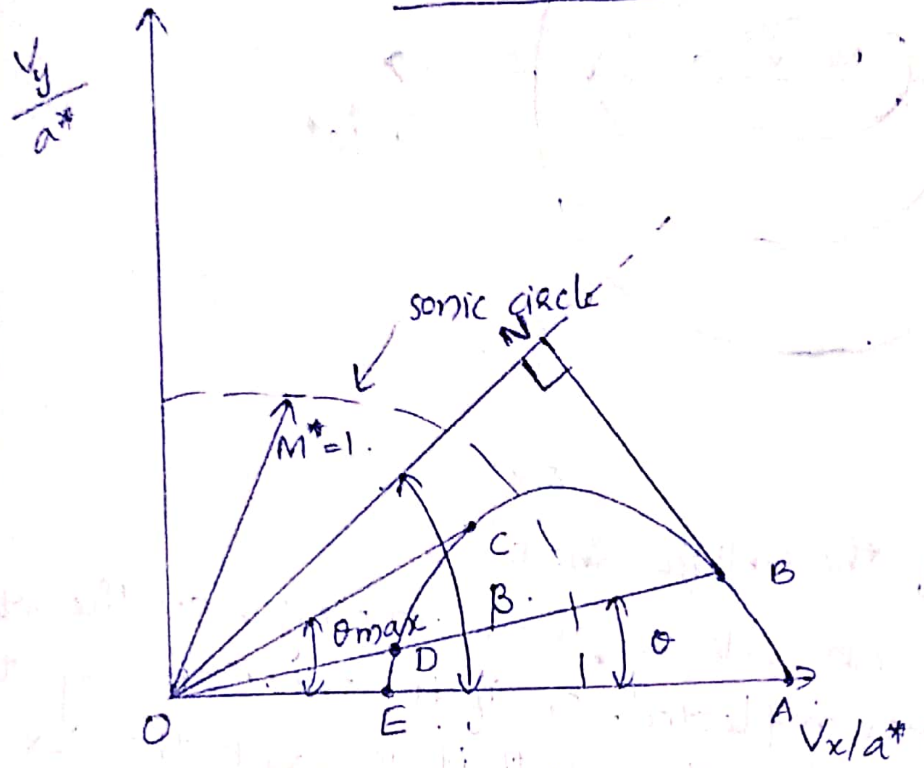
→ we know that flow across the shock is adiabatic. i.e., 'T' is a constant.

$a = \sqrt{\gamma R T}$

Here 'a' is a constant throughout.

→ The value of a^* is a constant upstream & downstream of the shock.

Dimensionless shock polar.



Circle with radius $M^* = 1$ is called sonic circle.

→ Circle with radius $M^* = 1$ is called sonic circle.

→ All the velocities inside the circle are subsonics.

→ All the velocities outside the circle are supersonics.

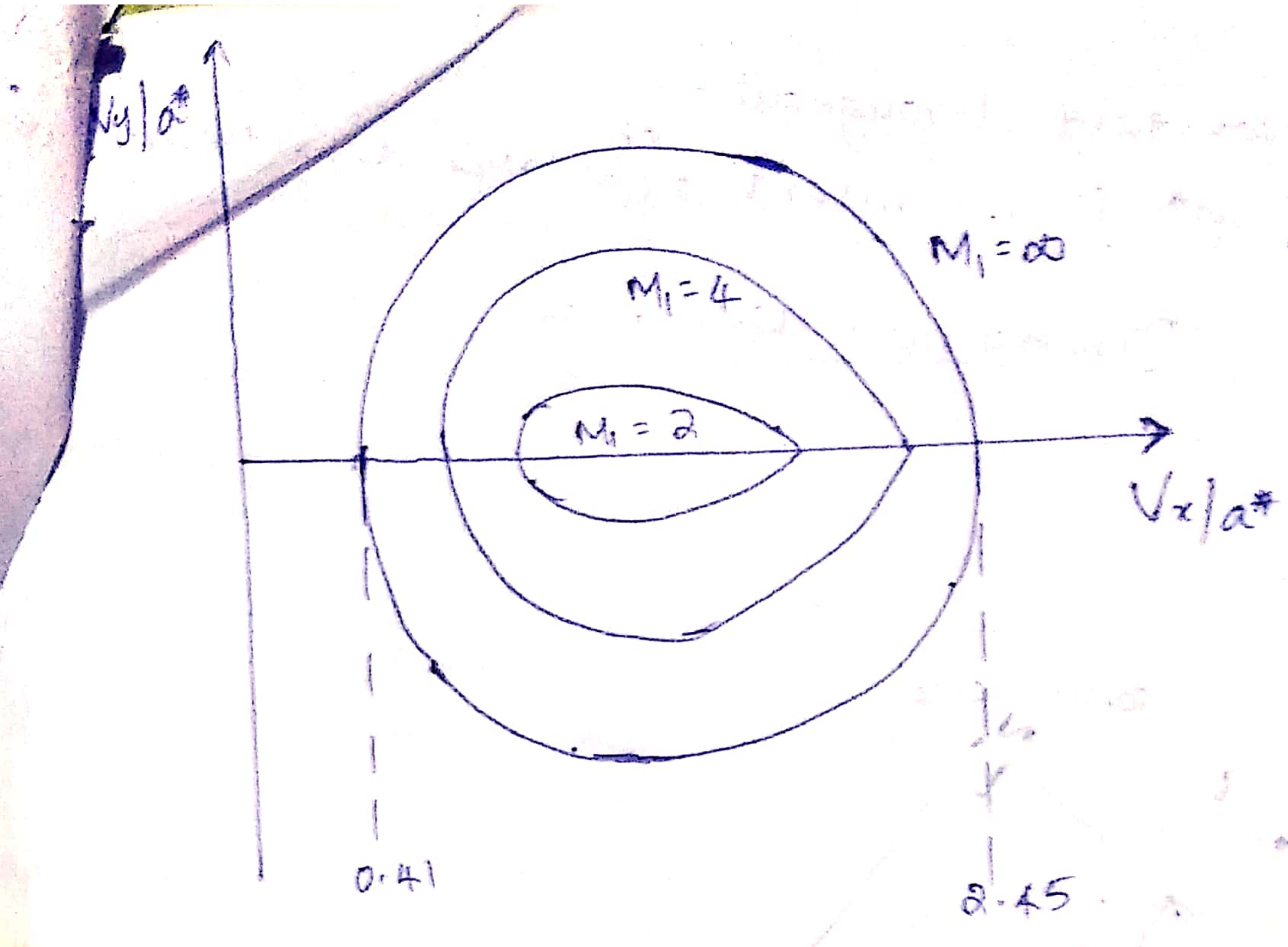
→ The shock polar can be described by an analytic eqn called the shock polar eqn.

$$\boxed{\left[\frac{V_y}{a^*}\right]^2 = \frac{\left(M_1^* - V_x/a^*\right)^2 \left[\left(\frac{V_x}{a^*}\right) M_1^* - 1\right]}{\frac{2}{\gamma+1} M_1^{*2} - \left[\frac{V_x}{a^*}\right] M_1^* + 1}}$$

→ The shock polars for different mach numbers form family of curves as shown in figure.

→ for $M_1^* = 2.45$ $(M_1 \to \infty)$; the shock polar is a circle.

Graph:



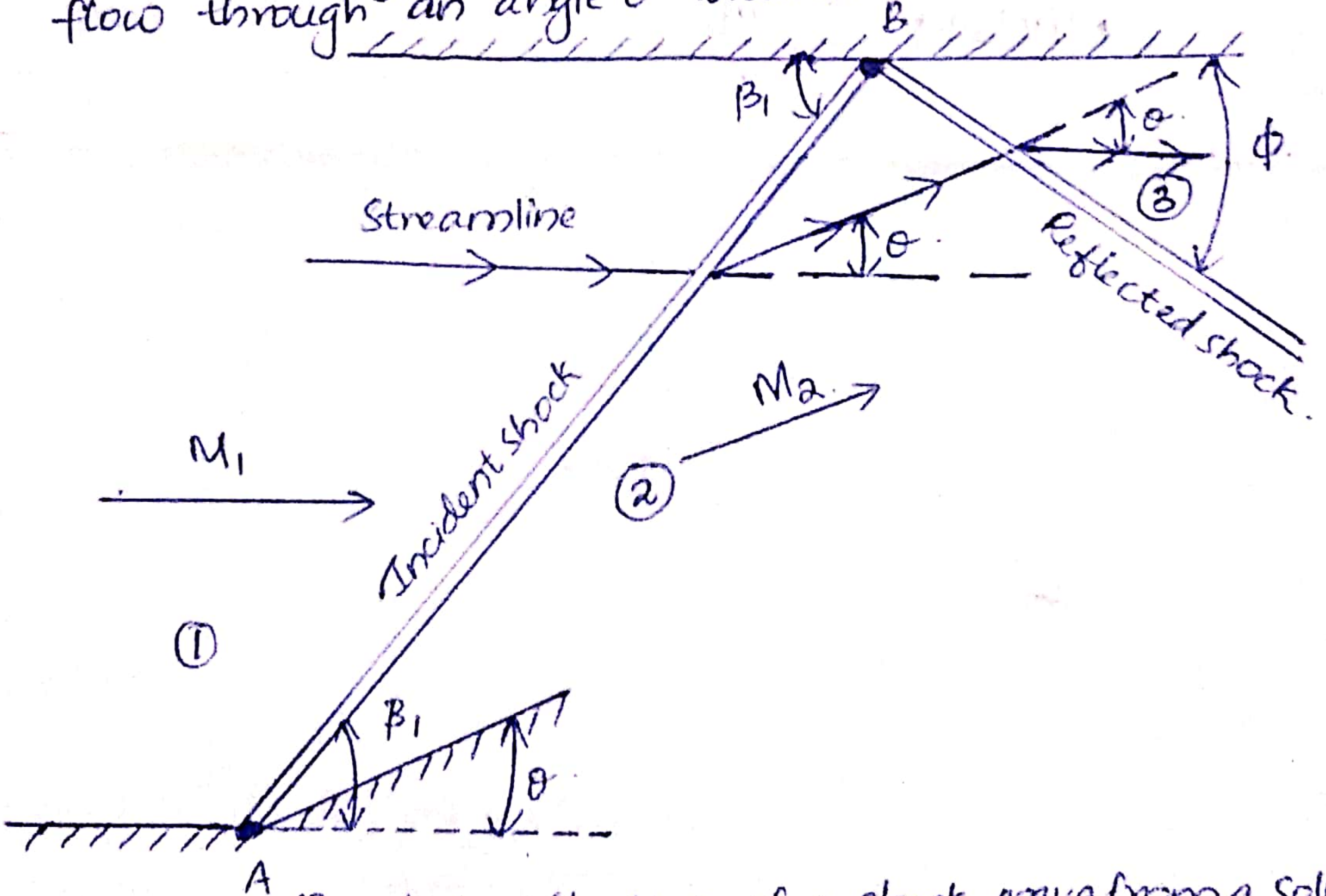
Vy/a*
M₁=∞
M₁=4
M₁=2
Vx/a*
0.41
2.45

* **Reflection of the Oblique Shock:**

When an oblique shock impinges on the solid surface, it gets reflected. If it strikes on any other waves it will also gets reflected.

If the incident shock is sufficiently weak, the reflection would be regular and could be treated according to linear theory, giving a reflected shock of the same strength as the incident shock.

If a shock which is not necessarily weak gets impinged on the wall, then the strength of the incident and the reflected shock may not be the same. The incident shock deflects the flow through an angle 'θ' towards the wall.



• Regular reflection of a shock wave from a solid boundary.

I In the above figure, A is the concave corner. Thus it generates an Oblique shock wave. The deflection angle at this corner is 'θ', 'β_1' is the angle between incident shock and the horizontal. A straight and a horizontal wall is present above this corner. The incident shock impinges on this upper wall at the point B.

In region ① the flow is streamline ie, 'M_1' is horizontal. After the shock the direction of the flow is varied. Since the flow should be tangential to the wall 'M_2' is inclined upwards making an angle 'θ' with the horizontal. At point B we get another shock which is known as the reflected shock. It is deflected downward by an angle 'ϕ'. The purpose of this reflected shock is to reflect the flow by keeping the angle 'θ' same. There will be difference in the strength of the incident and reflected shockwave. The strength of the reflected shock will be less than that of the incident shock.

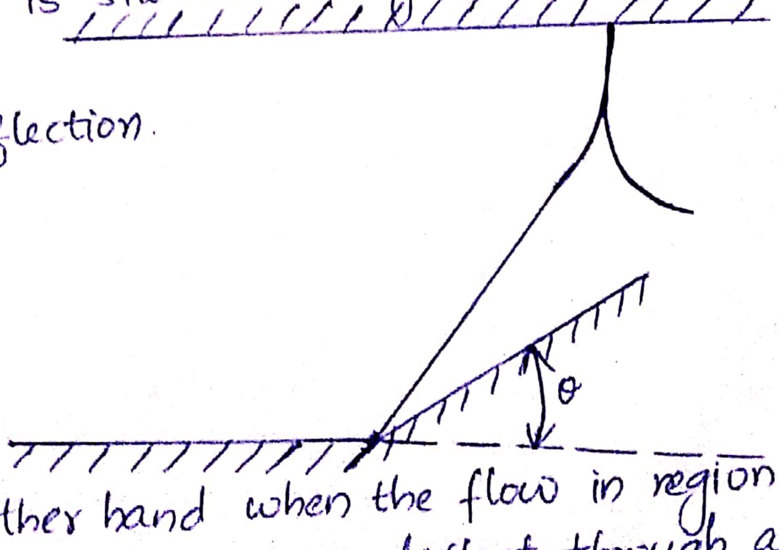
ie., $M_2 < M_1$ and $M_3 > M_2$.

Angles 'β' and 'ϕ' will be different since their strengths are different. (ie, $\beta \neq \phi$)

II We can calculate M_2 by considering M_1 and θ and M_3 can be calculated by considering M_2 and θ.

→ From the incident shock whether upstream mach number of M_1, $\theta < \theta_{max}$ and hence the incident shock is an allowable straight oblique shock solution. This straight incident shock is sketched in figure below:

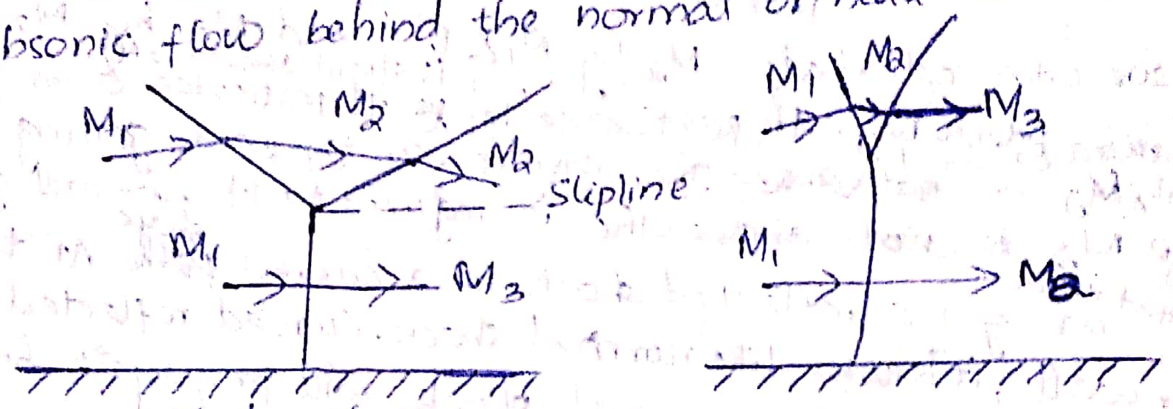
⊙ Mach reflection.



On the other hand when the flow in region 2 at mach number M_2 wants to again deflect through an angle 'θ' via the reflected shock, it finds that $\theta > \theta_{max}$ for M_2, and a

- There of regular reflection is not possible. Instead a normal shock is formed at the upper wall to allow the streamlines to continue parallel to the wall. Away from the wall, this normal shock terminate a curved shock which intersects the incident shock, with a curved reflected shock propogating downstream. This shock pattern is sketched in the figure and is labeled a mach reflection in contrast to the regular reflection.

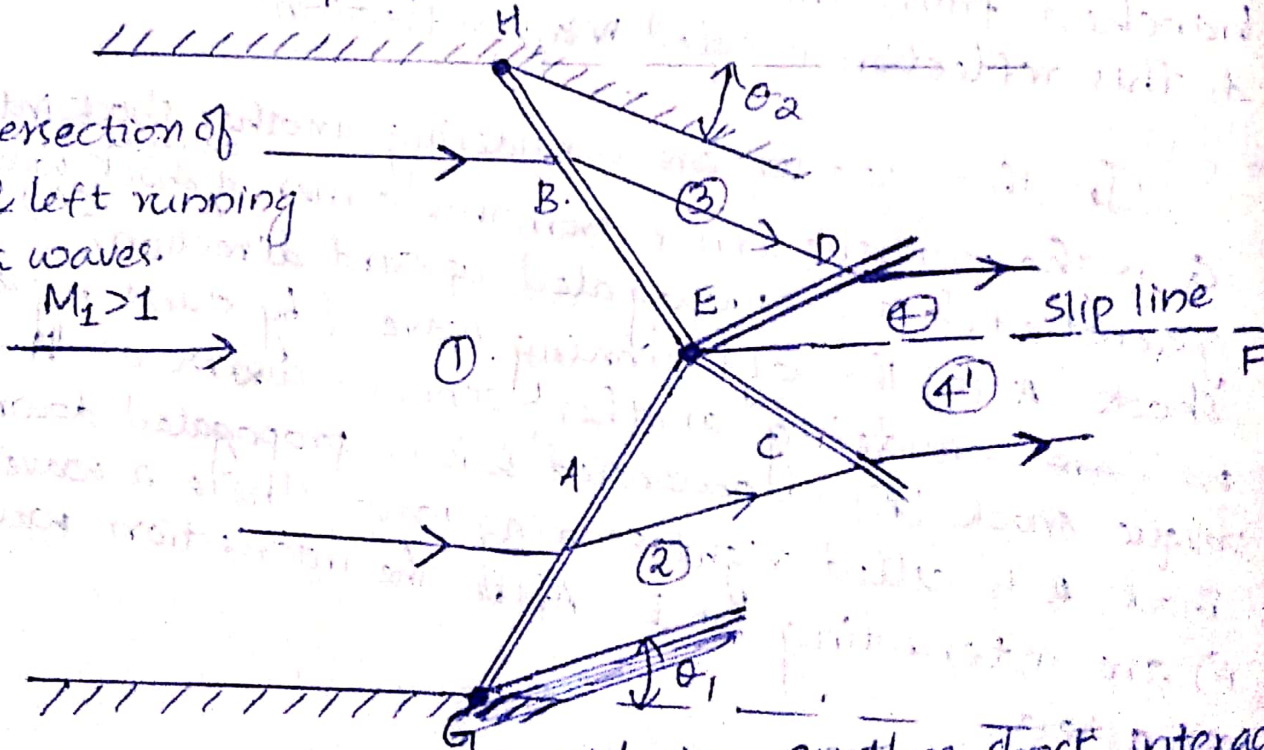
The mach reflection is characterised by large regions of subsonic flow behind the normal or near normal shocks.



Mach reflection

III

- Intersection of right & left running shock waves.



III In this case we are considering another shock interaction. Here 'G' is the concave corner. From this corner, a oblique shock wave is generated and it is propogated in upward direction. Shock A is the left running wave (by viewing from H). We are considering another concave corner at H. Another Oblique shock is generated and it is propagated in downward direction. shock B is called right running wave. These 2 waves A and B are intersecting at E. After the intersection wave A and B is refracted.

*Assignment:

1) The flow mach number, pressure & temperature ahead of a normal shock are given as 2, 0.5 atm & 300 K respectively. Determine mach number, pressure, Temperature & velocity behind the wave. ($\gamma = 1.4$)

2) If the entropy change caused by a normal shock in an airstream is 200 J/Kgk. Determine the mach number ahead of the shock and the shock strength.

3) A C-D Nozzle is designed to deliver mach 1.8 He stream. If this nozzle has to run with air with a normal shock at the exit, Determine the stagnation pressure required if the back pressure is 30KPa. Also find the Mach number ahead of the shock.

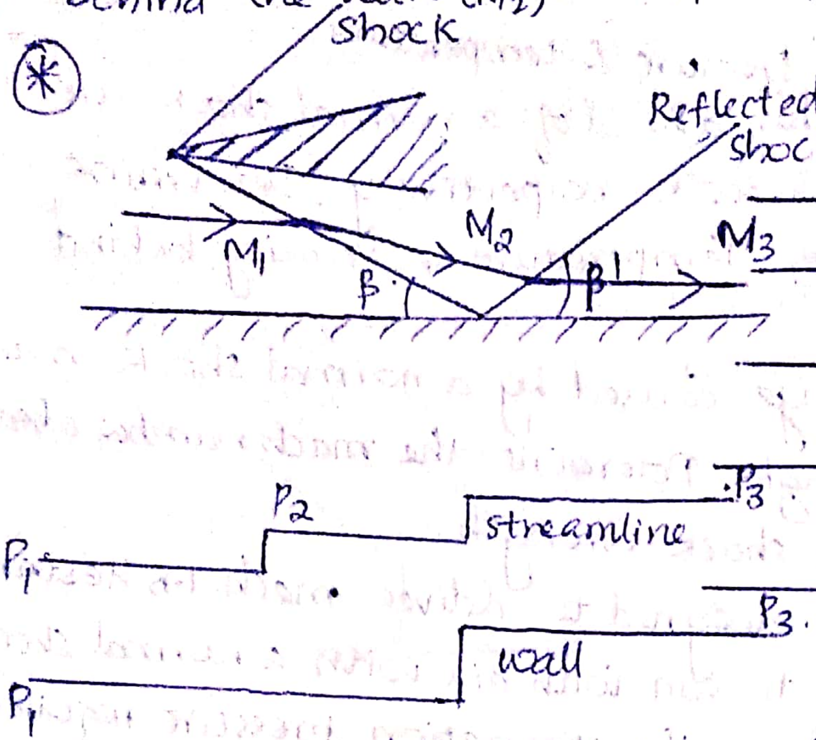
For He, $\gamma = 1.67$
$\frac{A_e}{A^*}$ will also be given in the table.

4) A gas flow with velocity 400m/s and specific volume 1.11 m³/kg passes through a normal shock. If the specific volume behind the shock is 0.333 m³/kg, Determine the pressure increase caused by the shock.

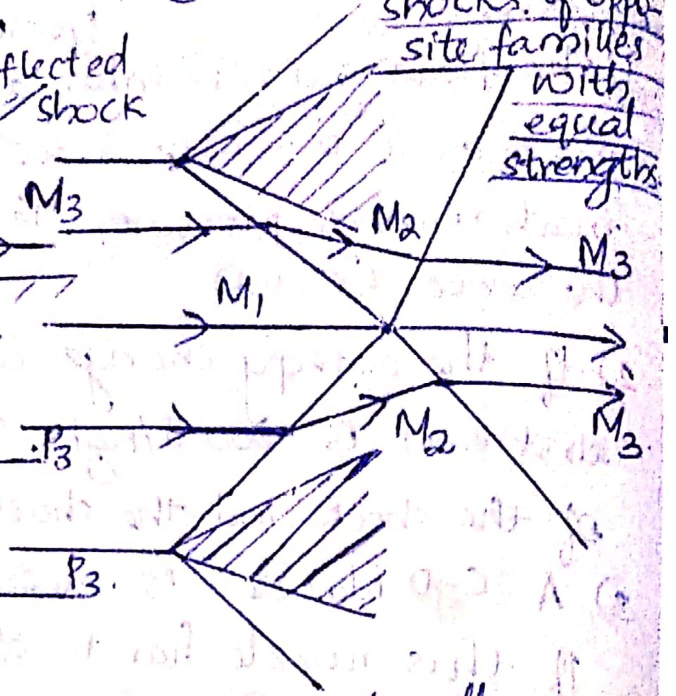
5) An oblique shock in air causes an entropy increase of 11.5 J/kgk. If the shock angle is 25°. Determine the 'M' ahead of the shock & the 'θ' (flow deflection angle).

6) Consider a Supersonic flow with M=2, P=1 atm & T=288 K. This flow is deflected at a compression corner through 20°. Calculate M, P, T, P_0 & T_0 behind the resulting oblique shock.

7) Consider an oblique shock wave with a wave angle of $30°$. The upstream flow (M_1) is 2.4. Calculate 'θ' of the shock wave & the flow, the P & T ratios across the shock wave $\frac{P_2}{P_1}$ & $\frac{T_2}{T_1}$ & the M behind the wave (M_2).



Interaction of 2 shocks of opposite families with equal strengths



Shock reflection from a rigid wall.

We know that for a shock which is not necessarily weak, the incident shock deflects the flow through an angle θ towards the wall. A second reflected shock of opposite family is required to turn the flow again by an amount θ, to satisfy the constraint of the wall. That is, the flow passing through the incident shock is deflected by an angle θ towards the wall as shown in figure. Therefore the reflected shock must deflect the flow in the opposite direction to the same angle θ, rendering the flow to travel parallel to the solid wall, in accordance with the flow physics that the streamline over a solid wall has to be parallel to the wall surface. Therefore, the mach number M_2 and wave angle of the reflected shock are dictated by the flow turning angle 'θ'.

Although the flow deflection 'θ' caused by the incident and reflected shocks is equal in magnitude, the pressure

ratios of the incident & reflected shocks are not equal since
$M_2 < M_1$. The pressure distribution along a streamline and
along the wall is shown in figure. The strength of the reflected
shock wave can be defined by the overall pressure ratio:

$$\frac{P_3}{P_1} = \frac{P_3}{P_2} \cdot \frac{P_2}{P_1}$$

where $\frac{P_3}{P_2}$ & $\frac{P_2}{P_1}$ are the strengths of reflected & incident
shocks respectively. Therefore the strength of reflection is
given by the product of individual shock strengths.

The reflection from a rigid surface is not specular ie.,
reflected shock inclination (β') is not equal to incident shock
inclination (β). Now there exists 2 possibilities: $\beta > \beta'$ or
$\beta < \beta'$.

These 2 cases are opposite & the net result depends on the
particular values of M_1 & θ. For higher Mach no:s (M_1) $\beta > \beta'$
whereas for low Mach no:s, $\beta < \beta'$. Reflection of a wave from
surface depends on the surface roughness. Reflection can be
either specular or diffuse. Specular reflection is that in
which the reflection is like a mirror image. ie., the wave
angles of incident & reflected waves will be the same. for
this surface should be smooth. But we know that smooth surf-
ace is only an imaginary situation. Rough surfaces cause
diffuse reflection. Thus, specular reflection is essentially an
assumption made to obtain simplified solutions for most prac-
tical problems.

→ Interaction of oblique shocks of opposite families with equal
strength:

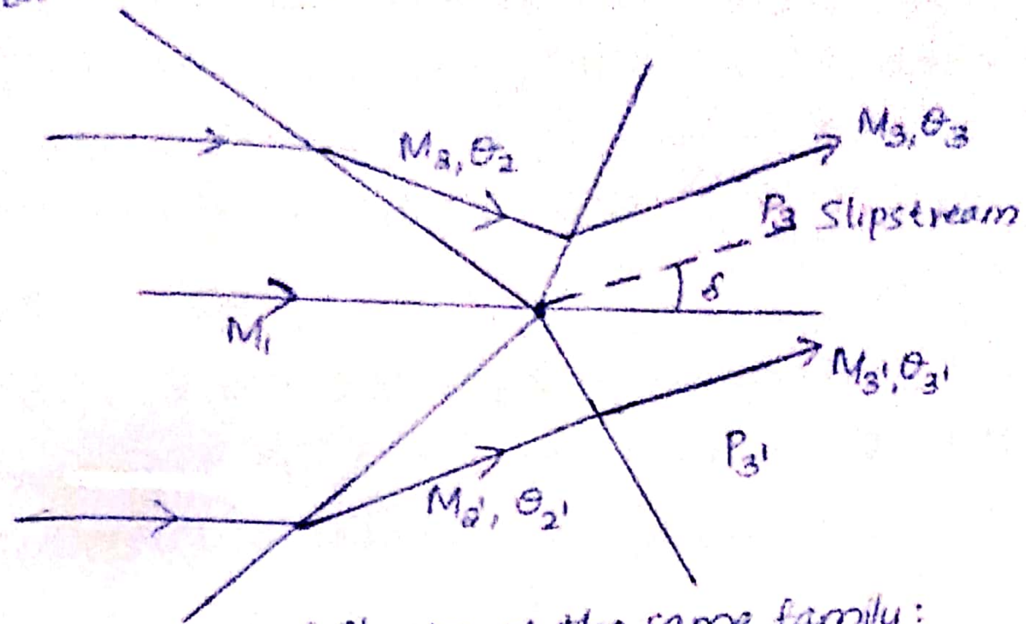
In this case the possible flow field is shown in
figure. Viewing in the flow direction, the shocks running
to the left are named as one family (the left-running
family) and the shocks running to the right are called the

Opposite family (the right running family).
The shocks "pass through" each other, but are slightly "bent" in the process. The flow downstream of the shock system is //el to the initial flow.

→ <u>Intersection of oblique shocks of opposite families with different strengths.</u>

when 2 shocks of unequal strength intersect, a new flow geometry is established as shown in figure. The flow field is divided into 2 portions by the streamline through the intersection point. The 2 portions experience different changes while passing through the shock wave system. The overall results must be such that, after crossing the wave system the 2 portions have the same pressure and same flow direction, ie, $P_3 = P_{3'}$ and $\theta_3 = \theta_{3'}$. The flow downstream of the reflected shocks (zone 3) need not be in the freestream direction. These 2 requirements determine the final direction δ and the final pressure P_3.

The streamline shown with the dashed line in figure, having 2 flow fields of different parameters (T & ρ) on either side of it, is called the contact surface. The contact surface can either be stationary or moving. It is idealized as a surface of discontinuity. Unlike the shockwave, there is no flow of matter across the contact surface. Contact surfaces are also called as material boundary, entropy discontinuity, slipstream or slip surface, vortex sheet and tangential discontinuity. Contact surface is a fluid boundary across which there is no mass ~~flow gradient~~ transport. This surface can tolerate temperature & density gradients but not pressure gradient. In other words, 'T' & 'ρ' on either side of the slipstream can be different but the 'P' on both sides are equal. It can also be stated as the contact surface can tolerate thermal & concentration imbalance but not pressure imbalance.

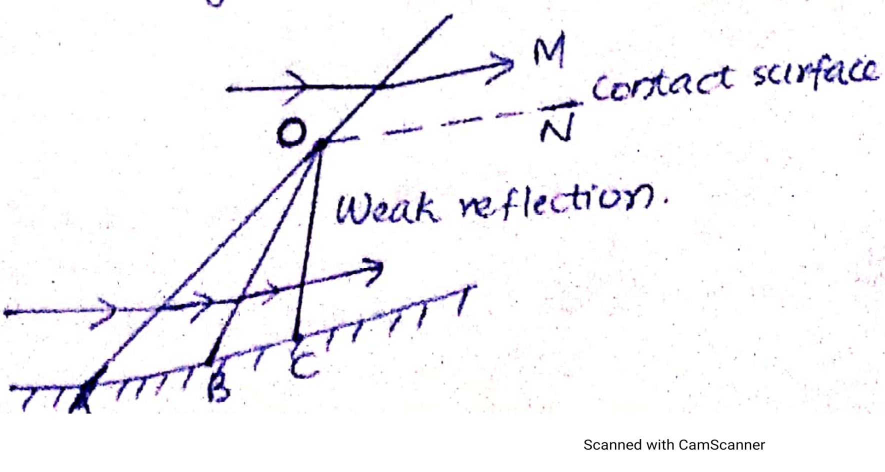


→ Interaction of Shocks of the same family:

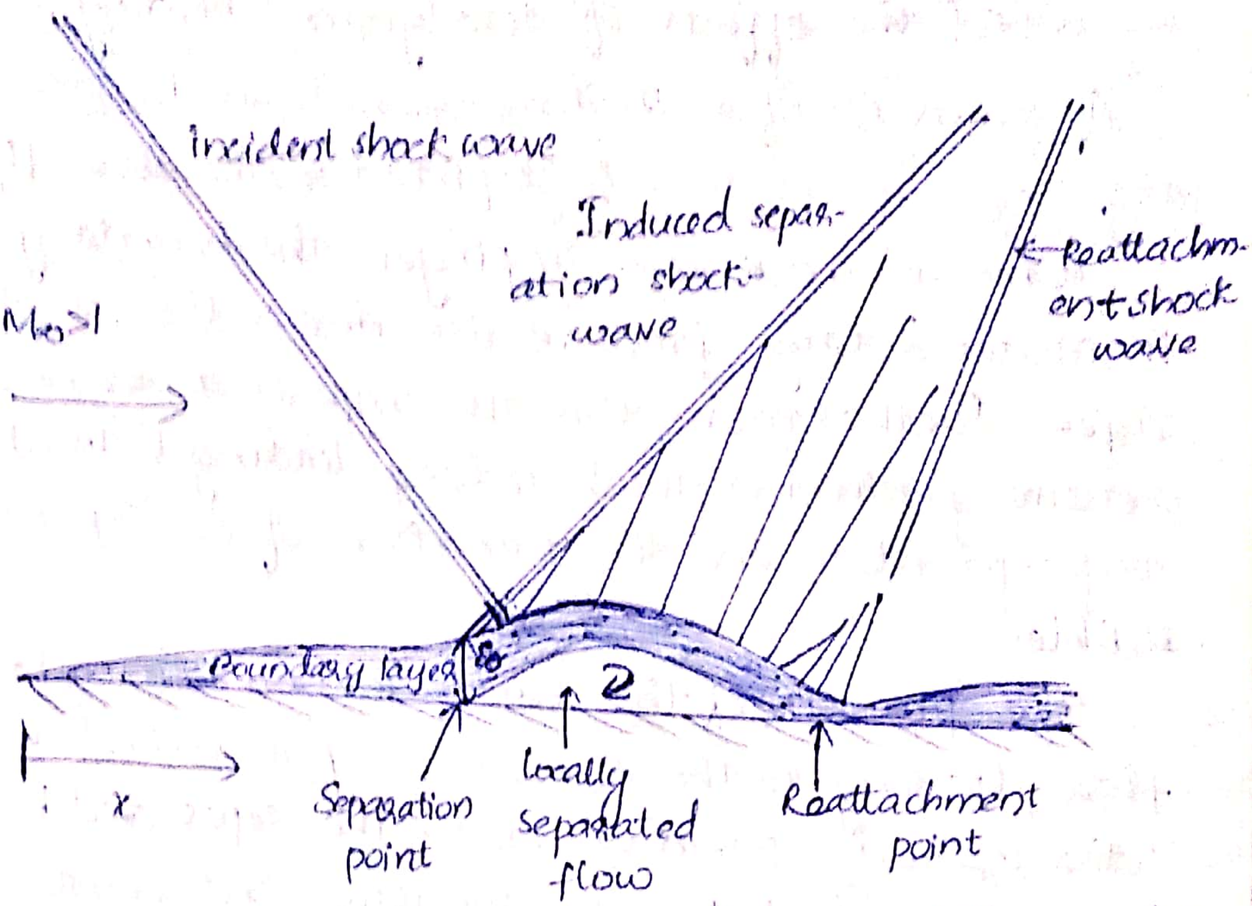
When a shock intersects another shock of the same family, the shocks cannot pass through as in the case of intersection of shocks of an opposite family. The shocks will coalesce to form a single stronger shock as shown in figures where shocks of the same family are produced by successive corners in the same wall.

If the second shock BO is much weaker than the 1st one (AO), then OC will be a compression wave. This intersection may be described as follows: The 2nd shock is partly transmitted along OM, thus augmenting the strength of the 1st one and partly reflected along OC.

Shocks are referred to as left-running and right running depending on whether they run to the left or right when viewed in flow direction. Left-running waves constitute one family & right-running waves constitute the opposite family.



* Shock wave and boundary layer interaction:



• Schematic of the shock-wave /boundary-layer interaction.

In a high speed flow, a shock wave can interact with another shock wave or immediately with the boundary layer, over the surface of an object.

Shock wave - boundary layer interactions are characterized by a number of phenomena namely:

(i) an incident shock.
(ii) an induced separation shock.
(iii) a reattachment shock.
(iv) an embedded expansion wave, and
(v) a separated flow region.

Shock-wave boundary layer interactions can cause severe aerodynamic heating that will result in extensive structural damage. Understanding these interactions can help flight

Vehicle designers to identify measures that would prevent or control the effects of aerodynamic heating.

A schematic of a 2-dimensional shock wave-boundary layer interaction is depicted in the above figure.

When a shock wave impinges the boundary layer, it causes a large pressure rise across the boundary layer. Furthermore, this also imposes a severe adverse pressure gradient on the boundary, leading to local boundary separation and the formation of a "separation bubble."

The presence of this bubble further disrupts the flow, leading to the formation of a second shock wave : the induced separation shock. The separated boundary layer turns back towards the plate, and eventually reattaches to the surface. When it does, the third shock wave (reattachment shock) is generated. The boundary layer is thinner, further downstream of the separation activity. Therefore, when the separated boundary layer reattaches, it encounters the thinner boundary layer, and because the pressure is still high, then the viscous interaction is high and so are the effects of aerodynamic heating. Further away from the plate, the separation and reattachment shocks merge to form the conventional reflected shock wave.

The scale and severity of the interaction depends on whether the boundary layer is laminar or turbulent. Laminar boundary layers separate more readily than turbulent boundary layers, so aerodynamic heating is more severe for laminar than turbulent interactions.